

REPORT DOCUMENTATION PAGE			Form Approved OMB NO. 0704-0188		
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1. REPORT DATE (DD-MM-YYYY)		2. REPORT TYPE Technical Report		3. DATES COVERED (From - To) -	
4. TITLE AND SUBTITLE An air MEDEVAC asset dispatching and prioritized casualty transporting model for military medical evacuation systems with distinguishable medical treatment facilities and errors in triage			5a. CONTRACT NUMBER W911NF-10-1-0176		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER 611102		
6. AUTHORS Benjamin Grannan, Laura McLay			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES Virginia Commonwealth University 800 East Leigh Street, Suite 3200 P.O. Box 980568 Richmond, VA 23298 -0568			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSOR/MONITOR'S ACRONYM(S) ARO		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S) 57352-NS.10		
12. DISTRIBUTION AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.					
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15. SUBJECT TERMS markov decision processes, military MEDEVACS, informational accuracy in decision-making					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Laura McLay
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 804-828-5842

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An air MEDEVAC asset dispatching and prioritized casualty transporting model for military medical evacuation systems with distinguishable medical treatment facilities and errors in triage

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February 13, 2014

Abstract

Decision making in military medical evacuation (MEDEVAC) of casualties consists of identifying which MEDEVAC asset to dispatch in response to a casualty and which medical treatment facility to transport the casualty, both of which contribute to the likelihood of casualty survival. These decisions become complicated when MEDEVAC assets and medical treatment facilities are distinguishable and casualties are prioritized as life-threatening and non life-threatening. In this paper, an undiscounted, infinite horizon Markov decision process model is developed that examines the interrelated decisions of how to optimally dispatch MEDEVAC assets to calls for service and transport casualties to medical treatment facilities. The model accounts for errors made during triage of casualties to investigate the revelation of information over time and allows for batch arrival of casualties to the system. The MDP is solved with a value iteration algorithm. The optimal policy is compared to three heuristic casualty transport policies.

Keywords: Markov decision process, military medical evacuation systems, triage

1 Introduction

Effective medical evacuation (MEDEVAC) of wounded soldiers (casualties) in military operations is important to the survivability of the combat soldier (Zinder, 2007). Transporting casualties to a medical treatment facility in a timely manner prevents the deteriorating health and potential death of casualties. The effective MEDEVAC of casualties also contributes to the potential psychological

advantage for those participating in combat, who understand that medical assistance will come quickly once requested (Bastian et al., 2012).

While this paper focuses on a model configuration for the United States, the model is applicable to other countries. Effective evacuation of casualties is an important problem shared across all countries that support combat troops. Moreover, issues examined in this paper, such as imperfect initial triage, the collection of new information over time, and medical guidelines for transporting military casualties, are generally shared across countries, as evidenced by a recent MEDEVAC summit held in London, England in October 2013 (MEDEVAC Summit, 2013).

This paper focuses on the dispatch and transport of casualties in United States military systems. Casualties arrive as calls for service, where dispatchers interpret the call detailing a casualty event and make a resource allocation decision regarding which MEDEVAC asset to dispatch to the casualty and later select which medical treatment facility to transport the casualty (Bozell, 2013). The MEDEVAC asset dispatched also transports the casualty to the medical treatment facility (i.e., a different MEDEVAC asset would not transport the casualty based on additional information collected at the scene) and therefore, the dispatch and transport decisions are interrelated.

Identifying effective policies for dispatching air MEDEVAC assets and transporting casualties can be counter-intuitive. A fleet of potential air MEDEVAC assets are distinguishable by their base location, and therefore articulate different response times to casualties. Likewise, medical treatment facilities are distinguishable by both the capable level of care, i.e., a role 2 medical treatment facility versus a role 3 medical treatment facility, and the proximity of the medical treatment facility to the casualty location. Further, there exists a triage scheme within military evacuation systems, in which the categories used to rank injuries for precedence in evacuation are as follows (Bozell, 2013):

- “CAT A”: Alpha category includes urgent casualties that need to be treated within one hour.
- “CAT B”: Bravo category includes priority casualties that need to be treated within four hours.
- “CAT C”: Charlie category includes routine casualties that need to be treated within twenty-four hours.

The evacuation triage system lends itself to sub-categorizing casualties based upon priority. For

example, a system that identifies only high-priority and low-priority casualties, the calls for service which have been categorized as CAT A could be seen as high-priority, while all other calls for service could be seen as low-priority. An alternative classification would be treating both the CAT A and CAT B calls for service as high-priority and the CAT C calls for service as low-priority. The prioritization scheme can be generalized for systems with more than two priority levels.

Military medical evacuation systems aim to transport CAT A casualties to a medical treatment facility within one hour, a practice commonly known as the Golden Hour (Bozell, 2013). The fundamental idea of the Golden Hour is that mortality is least likely to occur if initial treatment of a severe trauma casualty begins within one hour post injury. A military evacuation system is evaluated against the Golden Hour standard to increase survivability of the most urgent CAT A casualties. Improving the logistics of MEDEVAC systems to meet the Golden Hour standard is an important problem found frequently in the popular press (Pahon (2012); Doane (2011); Shinkman (2013)).

The Golden Hour performance measure evaluates time until treatment of a casualty, and it is in contrast with performance measures used by civilian emergency medical systems (EMS). Nearly all civilian EMS systems evaluate performance according to response times as opposed to casualty delivery times (McLay, 2010). As a result, nearly all research in civilian systems focuses on triage accuracy and initial dispatch decisions. However, the importance of triage on resource allocation decisions is well-documented area in civilian systems (see Clawson et al., 1999; Dunford, 2002), and this issue becomes more complex in military MEDEVAC systems because dispatch and transport decisions are interrelated and more accurate information is collected at the scene.

This paper formulates a Markov decision process (MDP) model to solve a MEDEVAC asset dispatching and casualty transporting problem with two interrelated types of decisions: first, how to initially dispatch location-dependent air MEDEVAC assets to location-dependent casualties, and second, how to identify distinguishable hospitals to transport the casualties. Both types of decisions indirectly affect the high-priority casualty’s likelihood of survival, which is dependent upon time until treatment in a medical treatment facility (Cunningham et al., 1997). To gain insight into military medical evacuation systems, the MDP model determines how to maximize the long-run average Golden Hour reward over the truly high-priority casualties while also providing

timely evacuation to low-priority casualties. The MDP model allows for classification errors in the initial triage, in which a truly low-priority casualty may be initially classified as high-priority, and vice-versa, thus leading to dispatch decisions with imperfect information. However, upon arrival at the scene, it is assumed that the medics from the responding air MEDEVAC asset accurately diagnose the severity of each call thus make transport decisions to the medical treatment facility with perfect information. The MDP model also accounts for batch, or multiple, casualties in a call for service.

This paper is organized as follows. Section 2 provides a literature review on military MEDEVAC asset optimization as well as dispatching and transporting models in the operations research literature. Section 3 outlines the novel MDP model. A computational example of the U.S. configuration is included in Section 4. Concluding remarks and future work are given in Section 5.

2 Background

There are a number of existing military research papers related to this effort. Higgins (2010) introduces the role and capabilities of U.S. Army MEDEVAC helicopters by providing an assessment of the operational issues. Operational issues of helicopters are pivotal in any research study of casualties and medical evacuation systems, due to the speed of response dictating survival likelihood. Several models focus on locating assets. Bastian (2010) presents a multi-criteria decision analysis model to determine the minimum number of MEDEVAC helicopters needed at each medical treatment facility to maximize the coverage of the theater-wide casualty demand, while minimizing the maximal medical treatment facility evacuation site total vulnerability to enemy attack. Zeto et al. (2006) also seeks to maximize the theater-wide casualty demand coverage, by examining the pre-location of air MEDEVAC assets, along with type and quantity, while balancing MEDEVAC asset reliability. Fulton et al. (2009) introduce a two stage stochastic optimization modeling framework for the medical evacuation of casualties, which identifies optimal casualty evacuation sites and medical treatment facility sites in response to stochastic demands for service. In contrast to asset emplacement strategy, this paper considers dispatching and transporting decision-making to maximize a Golden Hour utility function.

Bastian et al. (2012) examines the required capabilities of medical evacuation platforms of the future U.S. MEDEVAC platforms, including identifying the zero-risk aircraft ground speed. Bastian et al. (2013) further examine three research issues surrounding future U.S. MEDEVAC platforms, including optimal operational capabilities, trade-off considerations of different aircraft engines, and the effect of weaponizing the current MEDEVAC asset fleet on range, coverage radius, and response time. While Bastian et al. (2012, 2013) evaluate competing objectives of future casualty evacuation systems, this paper focuses on tactical issues such as real-time dispatch and transportation issues, two issues that have been overlooked in the military MEDEVAC optimization literature. Therefore, the model in this paper provides a unique contribution to the military MEDEVAC optimization literature by examining the inter-related dispatching and casualty transporting decisions given that there are errors in initial triage.

Military and civilian emergency service systems are similar in nature as both systems deal with the transportation of time-sensitive customers/patients to higher level medical care facilities. Further, both emergency service systems have high and low prioritized customers, a complexity that makes resource allocation decisions difficult. To improve response and transport times to the truly most critical patients, it is important to understand when to dispatch the closest server versus when to ration that asset instead. McLay and Mayorga (2013) present a MDP model for dispatching servers to spatially-distributed patients that maximizes the fraction of patients who are responded to within a fixed time frame while allowing for the possibility of classification errors in initial patient classification. This paper is similar to McLay and Mayorga (2013) as both develop a MDP model and consider potential classification errors in initial classification. However, they differ in that this paper evaluates the impact of additional information that becomes available over time during the response to and treatment of a casualty as well as its impact on transport decisions.

Several other papers have examined dispatch issues for civilian EMS and fire departments. Jarvis (1975) introduces a Markov decision process for determining optimal dispatching policies for a single type of server. Swersey (1982) develops a Markov model for determining how many fire engines to send to prioritized fire calls that balances the costs associated with dispatching too few or too many. Ignall et al. (1982) extend this model to account for calls and fire engines that are spatially distributed, and they provide a “preparedness” heuristic rather instead of exploring an optimal

solution. Both Andersson and Värbrand (2007) and Lee (2011) propose similar “preparedness” heuristics for dispatching ambulances to calls. A related stream of literature focuses on spatial queuing models and approximations that describe dispatching dynamics rather than prescribing dispatch decisions (Larson, 1974, 1975; Budge et al., 2009; Jarvis, 1985).

Emergency medical service systems identify which hospital to transport customers/patients. In the civilian side, the patient or protocols from the medical director dictate to which hospital an ambulance takes a patient, and therefore, there are rarely choices. Shunko et al. (2011) explores hospital transport decisions using game theory in the context of two competing hospitals that can send delay signals to turn away incoming ambulances, a situation that does not arise in military medical systems.

In summary, this paper is distinct from the existing civilian emergency service systems literature, because of its consideration of batch arrivals, prioritized casualties, and the inclusion of casualty transport in the modeling framework.

3 Markov decision process model

This section presents the MDP model for dispatching air MEDEVAC assets and transporting prioritized casualties in a military medical evacuation system with imperfect information during triage. The model parameters depend on the elapsed time during the treatment of each casualty, because military medical evacuation systems are evaluated by the transfer of high-priority casualties to the medical treatment facility before the Golden Hour. There are seven time steps to a military medical evacuation (Bastian, 2010):

1. Notification time (call arrival).
2. MEDEVAC asset departure time (“wheels up”).
3. Arrival at the scene.
4. Departure from the scene.
5. Arrival at the medical treatment facility.
6. Transfer of the casualty to medical treatment facility.
7. Arrival at MEDEVAC asset home location (return to service).

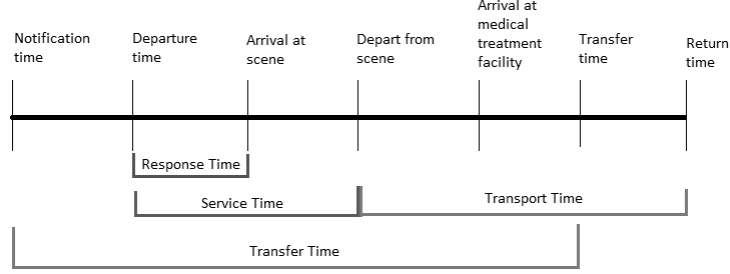


Figure 1: Time line during military medical evacuation

Figure 1 presents the four time intervals used throughout the remainder of this paper. *Response time* is the length of time from departure time (2) to the MEDEVAC asset arrival at the scene (3). *Service time* is the length of time from departure time (2) to leaving the scene (4). *Transport time* is length of time from the MEDEVAC asset leaving the scene (4) to returning to its home station after transporting a casualty (7). *Transfer time* is the length of time from injury (1) to the casualty being transferred to a medical facility (6).

The input parameters of the MDP model are summarized next, followed by the system dynamics.

n = the number of casualty locations,

m = the number of air MEDEVAC assets, each at a fixed home location,

d = the number of medical treatment facilities,

R = the *classified* risk level during triage, with $R \in \{H, L\}$, where H (L) denotes classified high-risk (low-risk),

r = the *true* risk level, with $r \in \{H', L'\}$, where H' (L') denotes truly high-risk (low-risk),

λ = the call arrival rate,

X = random variable representing the number of casualties $X \in \{1, 2, \dots, N\}$ arriving in a batch arrival,

P_i^X = the conditional probability that a batch call for service with X casualties arrives at location i , given that a call arrives, $i = 1, 2, \dots, n$, $X = 1, 2, \dots, N$,

$P_{R|i}^X$ = the conditional probability that a batch call for service with X casualties arrives at location i has classified risk level $R \in \{H, L\}$, given that a call arrives, $i = 1, 2, \dots, n$, $X = 1, 2, \dots, N$,

$P_{r|R \cap i}^X$ = the conditional probability that a batch call for service with X casualties and classified risk level $R \in \{H, L\}$ has true risk level $r \in \{H', L'\}$, $i = 1, 2, \dots, n$,

μ_{ij}^X = the expected service time when MEDEVAC asset j responds to a batch call for service with X casualties at location i , $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$, $X = 1, 2, \dots, N$,

δ_{ijk} = the expected transport time when MEDEVAC asset j transports a batch of casualties from location i to medical treatment facility k where $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$, and $k = 1, 2, \dots, d$,

u_{ijkr}^X = the expected utility when MEDEVAC asset j transports a batch of X casualties with true risk level $r \in \{H', L'\}$ from location i to medical treatment facility k , where $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$, $X = 1, 2, \dots, N$, and $k = 1, 2, \dots, d$.

The state variable reflects the positions of each of the m MEDEVAC assets and thus can be represented by the m -dimensional vector s , with $s(t) = (s_1, s_2, \dots, s_m)$. To describe the state space in a succinct way, we describe all possible values that each component of the state space can take. In state $s(t)$, MEDEVAC asset j has three possible types of values corresponding to the three possible events in the system 1) asset j can be sent to a call for service that arrives, while servicing this call at the scene, s_j is described by a call location i , a classified priority R , and a batch size X , 2) asset j finishes service at the scene of service and begins transporting casualties to a medical treatment facility denoted by D_k , and 3) a busy MEDEVAC asset becomes free and returns to its home location ($s_j = 0$). Note that these possible values for s_j — (i, R, X) , D_k , 0—are all distinct values that are mapped to integers in the computational implementation of the value iteration algorithm used to solve the model. The total number of states is equal to $(1 + 2Nn + nd)^m$. Although the MDP model suffers from the so-called “curse of dimensionality,” we will explore conditions under which the state space can be made smaller in Section 4.

Only one component of the state variable changes after an event occurs at time t . Therefore, $s(t+1) = s(t)$ except for component s_j in $s(t+1)$. Let ϕ denote the new value of s_j in state $s(t+1)$, either (i, R, X) , D_k , or 0. Let the transition function $s(t+1) = S^M(s(t)|s_j = \phi)$ capture the new state at time $t+1$.

The following assumptions are made in the model. First, if a call for service arrives, an available MEDEVAC asset must be dispatched to the casualty if any are available. Otherwise the call is assumed lost to our system. This assumption is acceptable because in practice, military systems leverage other assets to treat these casualties (Bozell, 2013). Second, service cannot be preempted and air MEDEVAC assets cannot be rationed in expectation of in-coming calls for service. Third, a MEDEVAC asset selects a medical treatment facility destination immediately prior to departing from the scene, and immediately after reassessing the casualty risk to obtain the true risk level r . Therefore, transportation of casualties is made with information of the true risk level $r \in \{H', L'\}$. This can be contrasted with the dispatch decision which is made with the potentially inaccurate triage classification. Thus, the interrelated decisions of dispatch and transport capture the revelation of information of each casualty's risk level over time. Fourth, the MEDEVAC asset that responds to the casualty must transport the casualty to the medical treatment facility. Fifth, batch arrivals of casualties at a location can be transported by a single responding MEDEVAC asset, that is the capacity of an air MEDEVAC asset is greater than or equal to the number of casualties in a batch. This assumption is reasonable, since in practice, the capacity of MEDEVAC asset is larger enough to transport virtually all batched casualties that arrive (Bastian, 2010). Lastly, risk levels are assessed on a batch level, not a casualty level. A single asset responds to a batch, not individual casualties, and therefore, risk levels assigned on the batch level is practical and easier to operationalize.

The objective of the MDP model is to determine which MEDEVAC asset to dispatch to a casualty and identify which medical treatment facility to transport a casualty, for each state in the state space, so that the expected number of truly high-priority calls that arrive at a medical treatment facility within one hour per stage is maximized.

The optimality equations for the infinite-horizon, average cost MDP model are given next, where $I_{\{s_j=(i,R,X)\}}$, $I_{\{s_j=D_k\}}$, and $I_{\{s_j=0\}}$ are indicator functions representing MEDEVAC asset j is serving X casualties at location i , traveling to medical treatment facility k , and being idle, respectively. An infinite-horizon MDP model with steady-state parameters is appropriate because of the duration of military operations. We use uniformization to convert a continuous time MDP model into an equivalent discrete time MDP model. To apply uniformization, the maximum rate of transitions is

determined to be $\gamma = \lambda + \sum_{j=1}^m \beta_j$, where $\beta_j = \max \left\{ \max_{i,X} \left\{ \frac{1}{\mu_{ij}^X} \right\}, \max_{i,k} \left\{ \frac{1}{\delta_{ijk}} \right\} \right\}$, $j = 1 \dots m$. Note that g is the optimal average utility per stage and $\nu_t(s(t))$ is a relative value function in state $s(t) = (s_1, s_2, \dots, s_m)$, and $A_1(s(t))$ and $A_2(s(t))$ represent the set of dispatching and transporting actions available in state $s(t)$ during iteration t , respectively.

$$g + \nu(s(t)) = \frac{1}{\gamma} \left[\sum_{k=1}^d \sum_{j=1}^m \sum_{i=1}^n (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} \nu(S^M(s(t)|s_j=0)) \right] \quad (1a)$$

$$+ \sum_{i=1}^n \sum_{X=1}^N \sum_{R \in \{H,L\}} \lambda P_i^X P_{R|i}^X \max_{j \in A_1(s(t))} \{ \nu(S^M(s(t)|s_j=(i,R,X))) \} \quad (1b)$$

$$+ \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} \sum_{r \in \{H',L'\}} (\mu_{ij}^X)^{-1} I_{\{s_j=(i,R,X)\}} P_{r|R \cap i}^X \max_{D_k \in A_2(s(t))} \{ \nu(S^M(s(t)|s_j=D_k)) + \gamma u_{ijk}^X \} \quad (1c)$$

$$+ \left(\gamma - \lambda - \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} (\mu_{ij}^X)^{-1} I_{\{s_j=(i,R,X)\}} - \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^d (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} \right) \nu(s(t)) \quad (1d)$$

The four lines in the value functions (1) represent the three events that result in a change in the state variable plus the fourth “event” that nothing changes between stages (line (1d)). The first line (1a) accounts for the event of busy air MEDEVAC assets completing service after transporting a casualty, where $(\delta_{ijk})^{-1}$ captures the mean transport time and $\nu(S^M(s(t)|s_j=0))$ represents the value of the state when MEDEVAC asset j returns home and is available for service. Line (1b) accounts for the dispatch of a MEDEVAC asset to a batch of casualties of size X that arrives to the system with classified priority R . Here, $\lambda P_i^X P_{R|i}^X$ captures the probability of a call for service of X casualties at casualty location i and of classified risk level R arrives to the system. The second part of line (1b) selects the MEDEVAC asset j to the incoming call for service (i, R, X) that maximizes the value of dispatching a MEDEVAC asset. Line (1c) accounts for the decision to transport a batch of casualties to a medical treatment facility, which occurs when service at the scene is completed. Here, $(\mu_{ij}^X)^{-1}$ represents the mean service time at the scene, and $P_{r|R \cap i}^X$ captures the conditional probability of the casualty’s true risk level r given its classified as risk level R and location i . The second part of line (1c) selects the medical treatment facility that maximizes the value of transporting casualties to a medical treatment facility, where u_{ijk}^X represents the reward received when MEDEVAC asset j transports X casualties at casualty location i of true risk level r to medical treatment facility k . Appendix A summarizes a value iteration convergence algorithm

using the corresponding N -stage finite-horizon optimality equations (see Puterman, 1994).

Lastly, the assessment of the severity of the calls is imperfect during triage, resulting in possible mismatches between the classified risk level $R \in \{H, L\}$ and the true risk level $r \in \{H', L'\}$ as captured by the $P_{r|R \cap i}$ parameters. The accuracy of triage classification is assumed known, such as from past system performance data. Since military medical evacuation systems are evaluated according to the response to casualties that are truly the most critical (H'), it is of interest to match the classified risk levels R to the true risk levels r . Let α denote the ratio of the proportion of classified high-risk casualties that are truly high-risk to the proportion of classified low-risk casualties that are truly high-risk,

$$\alpha^X = \frac{P_{H'|H}^X}{P_{H'|L}^X}.$$

Therefore, α^X can be interpreted as the accuracy of the triage for high-priority casualties, which we assume is independent of the casualty location. When $\alpha^X = 1.0$, the classified high-risk casualties are at least as likely to be truly high-risk as classified low-risk casualties. As $\alpha^X \rightarrow \infty$, the set of truly high-risk casualties is a subset of classified high-risk casualties (when $P_{H'} \leq P_H$). In the MDP model, input parameter $P_{r|R \cap i}^X$ is a function of α^X , and can be computed as follows. Note that $\alpha^X = \frac{P_{H'|H \cap i}^X}{P_{H'|L \cap i}^X}$ since triage accuracy is independent of the casualty location. Rearranging and applying Bayes rule yields:

$$P_{H'|H \cap i}^X = \frac{\alpha^X P_{H'}^X}{P_{L \cap i}^X} (P_{i|H'}^X - P_{H \cap i|h}^X).$$

Rearranging, noting that $P_{L \cap i|H'}^X + P_{H \cap i|H'}^X = P_{i|H'}^X$, and applying Bayes rule again yields:

$$P_{H'|H \cap i}^X = \frac{\alpha^X P_{H'|i}^X}{P_{H|i}^X + (1/\alpha^X) P_{L|i}^X}.$$

The analogous procedure can be applied to classified low-risk calls, yielding $P_{H'|L \cap i}^X$.

Appendix B contains theoretical results related to transportation policies in the MDP model proposed in this paper. The first result indicates that if the expected times to transfer a casualty at two medical treatment facilities are the same, it is optimal to transport to the medical treatment facility with the highest utility. The second result indicates that if the utilities for transferring a casualty at two medical treatment facilities are the same, it is optimal to transport to the medical

treatment facility with the shortest expected time until transfer. However, these results are not actionable when there are trade offs between transfer time and quality of care. Therefore, we examine the trade offs in the computational example in the following section.

4 Computational example

4.1 Problem setup

Consider a military medical evacuation system example in support of an U.S. Army brigade, where the location of casualties to be evacuated and medical treatment facilities are both known. As described in Bastian et al. (2012), the area of operations for future U.S. Army brigades (a military unit with over 3,000 personnel) is 300 km^2 , and a sub-area of 30 km^2 containing four air MEDEVAC assets and four casualty locations ($m = n = 4$) is proposed for analysis here (see Figure 2).

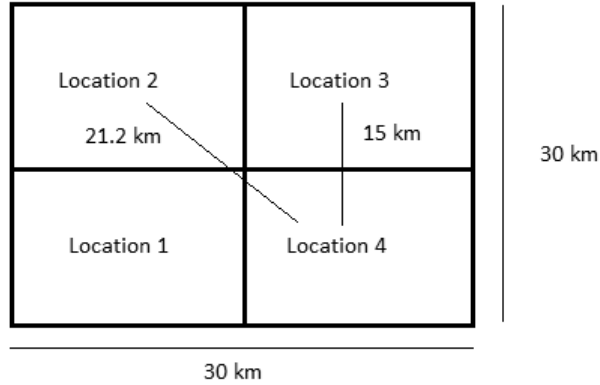


Figure 2: Geography of military medical evacuation system with 4 casualty locations and 4 air MEDEVAC assets

Each square in Figure 2 is 15 kilometers long and 15 kilometers wide. Travel times are computed using the Euclidean distance between locations and MEDEVAC assets have a flight speed of 155 nautical miles per hour (knots) Bastian (2010).

There are two distinguishable medical treatment facilities (i.e., $d = 2$) available in support of the Army brigade—the first is a role 2 medical treatment facility denoted $k = 2$, and the second is

a role 3 medical treatment facility denoted $k = 3$. The utilities $u_{i,j,k,H'}$ are set based on the time it takes for asset j to transfer a casualty at location i to medical treatment facility k . Suppose this transport time takes t hours, on average, to a role 2 medical treatment facility. Then, the modified Golden Hour utility function as a function of t is $\max\{\frac{-t}{60} + 1, 0\}$. The utilities for transporting a casualty to a role 3 medical treatment facility is assumed to be a factor of Γ increase over the role 2 utility. Moreover, we assume the utility for transporting true low-priority casualties is zero, since these casualties are expected to survive regardless of where they are transported.

We focus on the disparity in proximity and medical treatment quality between the role 3 and role 2 medical treatment facilities in this example. The relative utilities and travel times between these two types of facilities are pertinent when managing military medical evacuation system logistics. Therefore, define the *distance ratio* θ as the relative travel distance to the role 3 medical treatment facility as compared to the role 2 medical treatment facility for each call location i and responding MEDEVAC asset j :

$$\theta = \frac{\delta_{i,j,3}}{\delta_{i,j,2}}, i = 1, 2, \dots, n, j = 1, 2, \dots, m.$$

When $\theta = 2$, the relative transport time to the role 3 medical treatment facility is twice the relative transport time to the role 2 medical treatment facility, given the same MEDEVAC asset j and demand location i .

Define the *reward ratio* Γ to distinguish between the system utility received transporting a truly high-priority casualty to the role 3 medical treatment facility and the utility received transporting a truly high-priority casualty to the role 2 medical treatment facility.

$$\Gamma = \frac{u_{i,j,3,H'}}{u_{i,j,2,H'}}, i = 1, 2, \dots, n, j = 1, 2, \dots, m.$$

When $\Gamma = 2$, the role 3 medical treatment facility is twice as medically capable as the role 2 medical treatment facility, due to better resources, surgeons on staff, etc.

Table 1 reports the average utility when transporting to the role 2 medical treatment facility or the role 3 medical treatment facility, for each MEDEVAC asset j and casualty location i under the base case of the military medical evacuation system in this example.

Many of the transition probabilities depend on the length of time until a busy MEDEVAC asset

Table 1: Average utility when transporting a true high-priority casualty to the role 2 medical treatment facility and the role 3 medical treatment facility

$u_{i,j,2,H'}$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
$j = 1$	0.364	0.364	0.291	0.291
$j = 2$	0.312	0.417	0.312	0.269
$j = 3$	0.291	0.364	0.364	0.291
$j = 4$	0.312	0.343	0.312	0.343
$u_{i,j,3,H'}$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
$j = 1$	0.456	0.456	0.363	0.363
$j = 2$	0.390	0.520	0.390	0.336
$j = 3$	0.363	0.456	0.456	0.363
$j = 4$	0.390	0.429	0.390	0.429

becomes free, for each dispatching or transporting action, or a call for service ends. Table 2 presents the average service and transport time when responding to a call for service under the base case of the military medical evacuation system in this example.

Table 2: Average service times and average transport times (in hours)

$\mu_{i,j}$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
$j = 1$	0.500	0.552	0.574	0.552
$j = 2$	0.552	0.500	0.552	0.574
$j = 3$	0.574	0.552	0.500	0.552
$j = 4$	0.552	0.574	0.552	0.500
$\delta_{i,j,2}$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
$j = 1$	0.188	0.136	0.188	0.210
$j = 2$	0.136	0.083	0.136	0.157
$j = 3$	0.188	0.136	0.188	0.210
$j = 4$	0.210	0.157	0.210	0.231
$\delta_{i,j,3}$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
$j = 1$	0.376	0.271	0.376	0.419
$j = 2$	0.271	0.167	0.271	0.315
$j = 3$	0.376	0.271	0.376	0.419
$j = 4$	0.419	0.315	0.419	0.462

The remaining transition probabilities depend on the rate at which casualties arrive to the system. Calls arrive according to a Poisson process with parameter $\lambda = 3.0$ calls per hour. The distribution of calls P_i are unevenly spaced across the four casualty locations. There is a “hotbed” of activity and more frequent calls for service in location 2. Table 3 presents the base case set of input parameters and the corresponding ranges used for sensitivity analysis in this example.

All computations are performed on dual servers with Quad-Core 3.00 GHz processors and

Table 3: Input parameters and ranges considered for sensitivity analysis

Input parameter	Base case value	Parameter range
Medical treatment facilities d	2	-
Casualty locations n	4	-
Air MEDEVAC assets m	4	-
Classified risk levels R	2	-
Reward Ratio Γ	1.25	[1, 1.5]
Distance Ratio θ	2	[2, 6]
Triage accuracy α	10^6	[1, 10^6]
Call arrival rate per hour λ	3.0	[1.5, 3.25]
Casualties in a batch X	1	-
Probability of casualty at each location P_i	[0.25 0.50 0.10 0.15]	-

16GB RAM. To solve the Markov decision process model (see Puterman, 1994) a value iteration convergence algorithm with tolerance of 10^{-5} is used. The value iteration convergence algorithm is presented in Appendix A. The run time for the value iteration algorithm is approximately 250 minutes for the base case model with 83,521 states and 30 replications of a simulation for 10,000 casualties has a run time of approximately 18 minutes.

4.2 Policy Comparison

The optimal Markov decision process solution is compared to three heuristic policies: 1) transport all casualties to the most rewarding medical treatment facility 2) transport all casualties to the closest medical treatment facility and 3) transport low-priority casualties to the closest medical treatment facility and transport high-priority casualties to the most rewarding medical treatment facility. All three heuristics dispatch the closest available server. Figure 3 compares the objective function value of the MDP to the performance of the three heuristics. The objective function values are rescaled so that values reflect the average modified Golden Hour utility received per casualty. Insights to be gained from figure 3 include the magnitude of improvement in system performance from leveraging optimization techniques versus heuristic policies. The optimal policy yields solution values that are, on average, 4.55%, 21.32%, and 0.72% better than heuristics 1, 2, and 3, respectively, as the distance ratio θ increases. It is also of note that when the distance ratio is small, e.g. $\theta = 2$, the MDP only mildly outperforms heuristic 3 and heuristic 1. However, as the distance ratio θ increases, the margin in which the MDP outperforms the best heuristic increases.

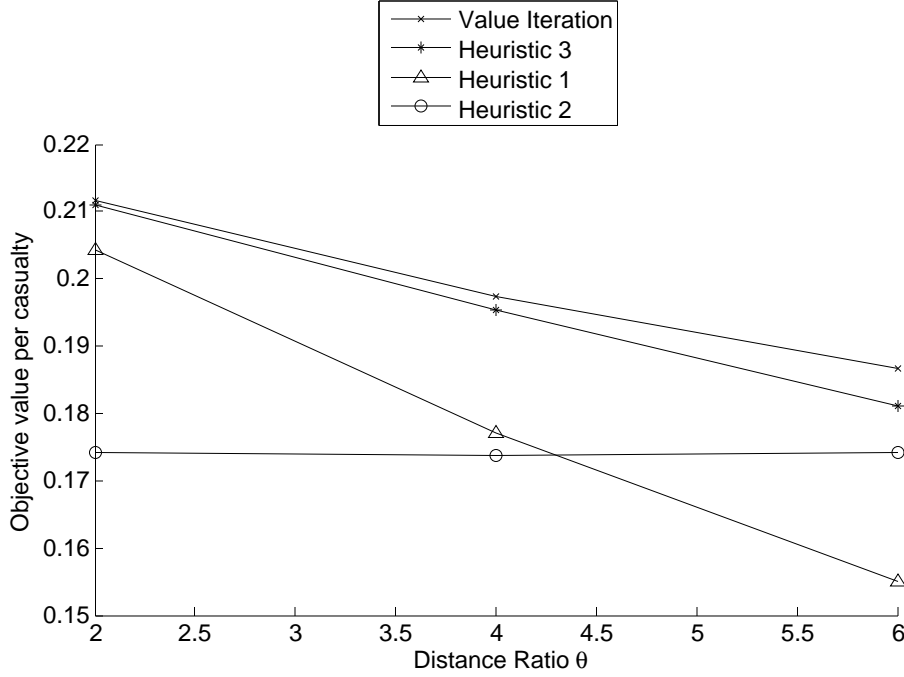


Figure 3: Simulated policy comparison

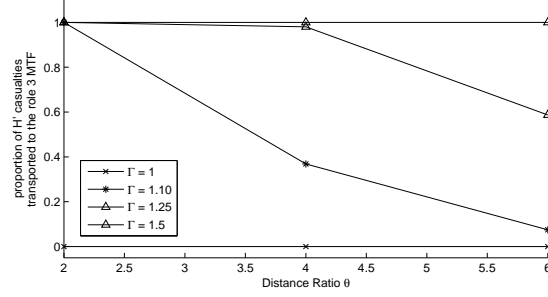
The loss rate is defined as the percentage of calls for service lost within the military medical evacuation system, due to the system being overcrowded with no available air MEDEVAC assets. For the MDP model, under the base case with $\lambda = 3.0$ and $\theta = 2$, the loss rate is 12.97%. Likewise, when $\theta = 6$ the loss rate is 16.38%. In practice, the so-called “lost” calls are delegated to non-traditional MEDEVAC assets so that all casualties receive timely service. Military medical evacuation systems differ from their civilian counterparts in that every effort is made to keep the queue for service at zero (Bozell, 2013).

4.3 Dispatching and Transporting Sensitivity

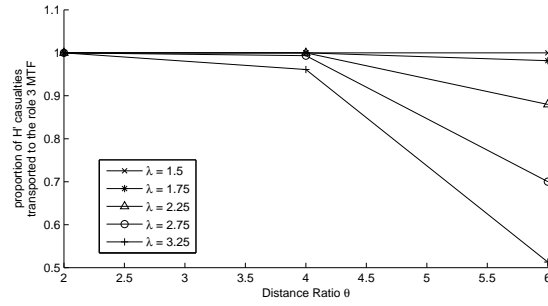
The MDP model optimizes over both the dispatching and transporting decisions, and therefore it is of interest to know when to transport casualties to the different medical treatment facilities. Figures 4(a) - 4(c) illustrate the sensitivity of the proportion of high-priority casualties delivered to the role 3 medical treatment facility as a function of the distance ratio θ for different Γ , λ , and α values. A main insight of military medical evacuation systems, seen in figure 4(a), is the impact

of the reward ratio Γ on the proportion of casualties transported to the more rewarding facility. Specifically, when the role 3 medical treatment facility is 50% better than the role 2 medical treatment facility ($\Gamma = 1.5$), all high priority casualties are transported to the more rewarding role 3 medical treatment facility, regardless of whether the role 3 medical treatment facility is close or far. Figure 4(b) provides insights on how the call arrival rate λ effects the system. An increase in λ floods the system with more casualties and more high priority casualties are transported to the role 2 medical treatment facility that is closer, so that servers can end service and be available to respond. It is optimal to deliver almost all true high-priority casualties to the role 3 medical treatment facility unless the role 3 medical treatment facility is very distant and there are few marginal benefits or high call volume. In sum, these results suggest that a heuristic that transports truly high-priority casualties to the medical treatment facility with the higher utility (unless it is extremely distant) and truly low-priority casualties to the nearest medical treatment facility, as done by Heuristic 3, can be used by military decision makers as a near optimal transportation policy. A heuristic transport policy has the added benefit of greatly reducing the state space to $(1 + nN)^m$ states, which helps to improve model scalability. However, it is less clear which asset to send upon initial dispatch. The dispatch decision is largely responsible for the difference in performance between the optimal policy and Heuristic 3 (see Figure 3), and therefore, we examine this issue next.

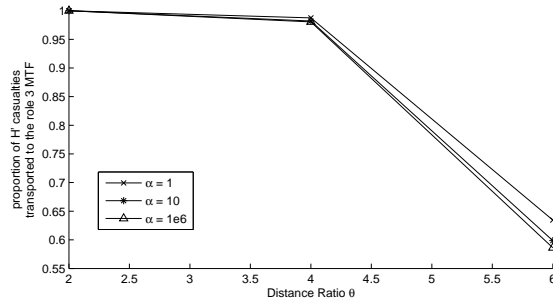
Table 4 presents the proportion of classified high-priority and low-priority calls to whom the closest MEDEVAC asset is dispatched, which captures system insights on whether to send the closest server or ration it instead. We note that the closest MEDEVAC asset is not always available, so it is impossible for these values to be equal to 1.0. We examine this decision across different levels of triage accuracy from a worst-case lower bound $\alpha = 1$ to $\alpha = 100$. Consider the classified H casualties in Table 4. The general insight we gain is that the frequency in which the closest server is dispatched decreases for locations 3 and 4, and increases for locations 1 and 2. The model is accounting for the call location distribution P_i , where location 1 and 2 have the greatest probability of a call for service, and reducing the response time to classified H calls for service in location 1 and 2. Reducing the response time by sending the closest server allows the servers to finish service and become available sooner for the additional calls expected in location 1 and 2.



(a) Γ sensitivity



(b) λ sensitivity



(c) α sensitivity

Figure 4: Sensitivity analysis on the proportion of true high-priority casualties H' transported to the role 3 medical treatment facility, with respect to distance ratio θ

In a similar manner, Table 4 also presents the closest MEDEVAC asset dispatching frequency for classified low-priority casualties. As α increases, each MEDEVAC asset generally responds to fewer calls in its “home” location. Therefore, when information is more accurate (i.e., when α is large), the system “saves” some MEDEVAC assets for responding to nearby truly high-priority casualties by strategically sending more distant MEDEVAC assets to low-priority casualties. This suggests that as classification accuracy improves, it is optimal to ration MEDEVAC assets in areas with the largest rate of truly high-priority calls.

Table 4: Proportion of calls for service at each location that are responded to by the closest MEDEVAC asset

θ	α	Location i	classified H calls	classified L calls
$\theta = 2$	$\alpha = 1$	1	0.506	0.506
		2	0.447	0.447
		3	0.503	0.503
		4	0.555	0.555
	$\alpha = 10$	1	0.516	0.416
		2	0.535	0.071
		3	0.460	0.456
		4	0.484	0.480
	$\alpha = 100$	1	0.534	0.256
		2	0.535	0.069
		3	0.461	0.430
		4	0.452	0.461
$\theta = 6$	$\alpha = 1$	1	0.434	0.434
		2	0.388	0.389
		3	0.422	0.422
		4	0.478	0.478
	$\alpha = 10$	1	0.447	0.349
		2	0.463	0.077
		3	0.409	0.406
		4	0.431	0.440
	$\alpha = 100$	1	0.457	0.237
		2	0.464	0.076
		3	0.415	0.347
		4	0.415	0.411

We further study the impact of the initial dispatch decision on later transport decisions by examining whether the transport decisions depend on the MEDEVAC asset dispatched. Recall that the MEDEVAC asset dispatched to a call for service also transports the casualty. Table 5 shows the proportion of true high-priority casualties that are transported to role 3 medical treatment

Table 5: Proportion of true high-priority casualties transported to role 3 medical treatment facility based on the responding MEDEVAC asset

θ	α	i	Closest MEDEVAC asset responds	More distant MEDEVAC asset responds
2	$\alpha = 1, 10, \text{ and } 100$	1	1.000	1.000
		2	1.000	1.000
		3	1.000	1.000
		4	1.000	1.000
6	$\alpha = 1$	1	0.541	0.249
		2	1.000	0.780
		3	0.809	0.231
		4	0.480	0.037
	$\alpha = 10$	1	0.560	0.139
		2	1.000	0.761
		3	0.775	0.145
		4	0.429	0.042
	$\alpha = 100$	1	0.563	0.105
		2	1.000	0.756
		3	0.751	0.107
		4	0.317	0.053

facility in two cases: when the closest MEDEVAC asset responds and when further MEDEVAC assets respond. Here, we see that when $\theta = 2$ all responding MEDEVAC assets transport casualties to the role 3 medical treatment facility, both when dispatch classification is poor ($\alpha = 1$) and when dispatch classification is better ($\alpha = 100$). Also, when the role 3 medical treatment facility is further and $\theta = 6$, we see that more distant responding MEDEVAC assets are less likely to later transport truly high-priority casualties to role 3 medical treatment facilities. There is less incentive (in terms of the Golden Hour performance measure) to transport casualties to the role 3 medical treatment facility when more distant MEDEVAC assets respond to a casualty. Tables 4 and 5 together shed light on how the revelation of information affects decisions throughout the treatment and delivery of each casualty. In particular, when there are more initial classification errors, distant assets are more likely to respond to H' casualties, who are then less likely to be transported to medical treatment facilities with the best capabilities.

5 Conclusions

This paper models and analyzes optimal dispatching and transporting policies in military medical evacuation systems. Timely transportation of casualties motivates the need to examine how to make better interrelated decisions—how to dispatch MEDEVAC assets to casualties and then transport casualties to medical treatment facilities—given the revelation of information over the duration of each call. An undiscounted, infinite horizon, average-cost MDP model is formulated to identify optimal policies, which is solved using a value iteration algorithm. In the computational example, a situation where two medical treatment facilities are distinguishable by both their proximity to calls for service (distance) and treatment capability (reward) is considered. Each dispatching and transporting decision effects system resources being busy or available to respond to additional calls for service. Optimal decision policies utilize the better role 3 medical treatment facility with varying frequency, as system input parameters such as call volume and dispatcher classification ability are varied. The optimal policy outperforms three heuristics considered in this paper on average by 4.55%, 21.32%, and 0.72%, respectively. The initial dispatch decisions account for much of the improvement over the heuristic policies. The computational results suggest that for most settings, a heuristic policy could be used for the transport decisions, which would greatly reduce the state space and improve model scalability.

Future work will focus on the locating of two types of MEDEVAC assets, such as air assets and ground assets. Another extension is to consider co-locating multiple types of dependent military assets, such as a MEDEVAC asset and a security escort asset, to dispatch one unit of each type in tandem to a casualty incident. A bi-objective model for balancing casualty Golden Hour coverage levels and risk tolerance, such as found in risky evacuation missions. Work is also under way to address these extensions.

Acknowledgments

The authors would like to thank the anonymous referees whose comments and suggestions greatly improved the manuscript. The authors would also like to acknowledge Captain Nathaniel D. Bastian, US Army, Mr. Jack Zeto, and Captain Joseph M. Bozell, USMC for general information

on military MEDEVAC logistics. Both authors were supported by the U.S. Department of the Army under Grant Award Number W911NF-10-1-0176. The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Department of the Army.

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Appendices

A Value Iteration Convergence Algorithm

To solve for the optimal policy, the relative value function algorithm (see Puterman, 1994) is run using the finite-horizon value functions. Therefore, t is the iteration here, not time. To do so, define $\nu_t(s(t))$ as the value of being in state $s(t)$ during iteration t , for $t = 0, \dots, N - 1$, and $\nu_0(s(t)) = 0$ for all $s(t) \in S$.

$$\nu_{t+1}(s(t)) = \frac{1}{\gamma} \left[\sum_{k=1}^d \sum_{j=1}^m \sum_{i=1}^n (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} \nu_t(S^M(s(t)|s_j=0)) \right. \quad (2a)$$

$$\left. + \sum_{i=1}^n \sum_{X=1}^N \sum_{R \in \{H,L\}} \lambda P_i^X P_{R|i}^X \max_{j \in A_1(s(t))} \{ \nu_t(S^M(s(t)|s_j=(i,R,X))) \} \right. \quad (2b)$$

$$\left. + \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} \sum_{r \in \{H',L'\}} (\mu_{ij}^X)^{-1} I_{\{s_j=(i,R,X)\}} P_{r|R \cap i}^X \max_{D_k \in A_2(s(t))} \{ \nu_t(S^M(s(t)|s_j=D_k)) + \gamma u_{ijk}^X \} \right. \quad (2c)$$

$$\left. + \left(\gamma - \lambda - \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} (\mu_{ij}^X)^{-1} I_{\{s_j=(i,R,X)\}} - \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^d (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} \right) \nu_t(s(t)) \right] \quad (2d)$$

To achieve the optimal policy, the relative value iteration algorithm is run until the upper and lower bounds converge to the optimal average utility per stage g

$$L_t \leq L_{t+1} \leq g \leq U_{t+1} \leq U_t,$$

with lower bound

$$L_t = \min_{s(t) \in S} [\nu_{t+1}(s(t)) - \nu_t(s(t))]$$

and upper bound

$$U_t = \max_{s(t) \in S} [\nu_{t+1}(s(t)) - \nu_t(s(t))].$$

The value iteration algorithm is executed until $U_{t+1} - L_{t+1} \leq \epsilon$, for a given ϵ .

B Theoretical Results

We consider the finite stage optimality equations and consider the limit. The N -stage case MDP equations are in Appendix A. Note that in section 3, equation 1a - 1d capture the exact, infinite horizon, average cost optimality equations. In contrast, equation 2a - 2d capture the optimality equations for the finite-horizon case.

Next we exploit the finite case optimality equations to analyze the MDP structural properties. The first lemma shows that it is always optimal to choose to transport casualties to the more rewarding medical treatment facilities when two when two medical treatment facilities have the same expected transport time. This suggests that transporting to the closest facility is optimal.

Lemma 1. *Suppose a MEDEVAC asset j finishes service at location i and needs to transport a casualty with risk level r to one of two medical treatment facilities, labeled as 1 and 2, $r \in \{H', L'\}$. If both facilities have the same expected transport time, i.e., $\delta_{ij1} = \delta_{ij2}$ and facility 1 has a higher utility than facility 2, i.e., $u_{ij1r}^X \geq u_{ij2r}^X$, then it is always better to deliver to the facility with the highest utility.*

Proof. Without loss of generality, assume that the system is in state $s(t)$ with value $\nu_t(s(t))$. The set of available transport decisions here are $A_2(s(t)) = \{D_1, D_2\}$, which correspond to facilities 1 and 2. Let $s_1(t+1)$ and $s_2(t+1)$ denote the states when facilities 1 and 2 are selected, respectively (see (1c)). It is sufficient to show that $\nu_t(s_1(t+1)) + \gamma u_{ij1r}^X \geq \nu_t(s_2(t+1)) + \gamma u_{ij2r}^X$. Note that in this case, the state in place j corresponding to asset j moves into the same transport state (i.e., $s_j = D$ whether medical treatment facility 1 or 2 is selected. Then, we can rearrange this to obtain

$$\gamma u_{ij1r}^X - \gamma u_{ij2r}^X \geq \nu_t(s_2(t+1)) - \nu_t(s_1(t+1)).$$

Since $\delta_{ij1} = \delta_{ij2}$ for $i = 1, \dots, n$, $j = 1, \dots, m$, then the value functions in these two states entirely cancel, yielding $\gamma u_{ij1r}^X - \gamma u_{ij2r}^X \geq 0$, which is true since $\gamma > 0$ and $u_{ij1r}^X \geq u_{ij2r}^X$ for $i = 1, \dots, n$, $j = 1, \dots, m$, $r \in \{H', L'\}$. \square

The next proposition shows that the average utility per stage is higher in a state when a MEDEVAC is available as compared to when it is busy transporting a casualty.

Proposition 1. Let state $s(t)$ be a state where server j is available, i.e., $s_j(t) = 0$. Let state $\hat{s}(t)$ be the corresponding state where server j is transporting a casualty, i.e., $\hat{s}_j(t) = D_k$ for some k and $\hat{s}_l(t) = s_l(t)$, $l = 1, \dots, m$ and $l \neq j$. Then $\nu_t(s(t)) - \nu_t(\hat{s}(t)) \geq 0$ for all $t \geq 0$.

Proof. The claim will be shown by induction. First, note that the claim is trivially true for $t = 0$ since $\nu_t(s(0)) = 0$ for all states $s(0)$. Let $s(t)$ and $s(t+1)$ denote identical states at different times. After some rearranging:

$$\begin{aligned}
& \gamma(\nu_{t+1}(s(t+1)) - \nu_{t+1}(\hat{s}(t+1))) = \\
& \sum_{k=1}^d \sum_{j=1}^m \sum_{i=1}^n (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} (\nu_t(S^M(s(t)|s_j=0)) - \nu_t(S^M(\hat{s}(t)|\hat{s}_j=0))) \\
& + \sum_{i=1}^n \sum_{X=1}^N \sum_{R \in \{H,L\}} \lambda P_i^X P_{R|i}^X \left(\max_{j \in A_1(s(t))} \{ \nu_t(S^M(s(t)|s_j=(i,R,X))) \} - \max_{j \in A_1(\hat{s}(t))} \{ \nu_t(S^M(\hat{s}(t)|\hat{s}_j=(i,R,X))) \} \right) \quad (3a) \\
& + \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} \sum_{r \in \{H',L'\}} \frac{P_{r|R \cap i}}{\mu_{ij}^X} I_{\{s_j=(i,R,X)\}} \left(\max_{D_k \in A_2(s(t))} \{ \nu_t(S^M(s(t)|s_j=D_k)) + \gamma u_{ijk}^X \} - \right. \\
& \quad \left. \max_{D_k \in A_2(\hat{s}(t))} \{ \nu_t(S^M(\hat{s}(t)|\hat{s}_j=D_k)) + \gamma u_{ijk}^X \} \right) \quad (3b) \\
& + \left(\gamma - \lambda - \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} (\mu_{ij}^X)^{-1} I_{\{s_j=(i,R,X)\}} - \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^d (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} \right) (\nu_t(s(t)) - \nu_t(\hat{s}(t))).
\end{aligned}$$

Note that in line (3a), the set of actions in $A_1(s(t))$ is a subset of those in state $A_1(\hat{s}(t+1))$. Let $j^* = \arg \max_j \{A_1(\hat{s}(t+1))\}$. We can bound the expression above from below by setting both decisions in (3a) to j^* . Likewise, we can apply this same idea to the actions in $A_2(s(t))$ selected in both maximizations in (3b). Let $d^* = \arg \max_{D_k} \{A_2(\hat{s}(t+1))\}$, and set the destination in the first maximization to d^* . Then,

$$\begin{aligned}
& \gamma(\nu_{t+1}(s(t+1)) - \nu_{t+1}(\hat{s}(t+1))) = \\
& \sum_{k=1}^d \sum_{j=1}^m \sum_{i=1}^n (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} (\nu_t(S^M(s(t)|s_j=0)) - \nu_t(S^M(\hat{s}(t)|\hat{s}_j=0))) \\
& + \sum_{i=1}^n \sum_{X=1}^N \sum_{R \in \{H,L\}} \lambda P_i^X P_{R|i}^X \left(\nu_t(S^M(s(t)|s_{j^*}=(i,R,X))) - \nu_t(S^M(\hat{s}(t)|\hat{s}_{j^*}=(i,R,X))) \right) \\
& + \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} \sum_{r \in \{H',L'\}} \frac{P_{r|R \cap i}}{\mu_{ij}^X} I_{\{s_j=(i,R,X)\}} \left(\nu_t(S^M(s(t)|s_j=d^*)) - \nu_t(S^M(\hat{s}(t)|\hat{s}_j=d^*)) \right) \\
& + \left(\gamma - \lambda - \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H,L\}} (\mu_{ij}^X)^{-1} I_{\{s_j=(i,R,X)\}} - \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^d (\delta_{ijk})^{-1} I_{\{s_j=D_k\}} \right) (\nu_t(s(t)) - \nu_t(\hat{s}(t))).
\end{aligned}$$

Here, all four lines are non-negative using by the induction assumption. Therefore, the claim is true. □

The second lemma shows that it is always optimal to choose to transport casualties to the “closer” medical treatment facility if both facilities have the same utility, where a facility is “closer” to a casualty at location i with asset j if its expected transport time is smaller,

Lemma 2. *Suppose in state $s(t)$ a MEDEVAC asset finishes service at the scene and needs to transport a casualty with risk level r to one of two medical treatment facilities, labeled as 1 and 2, $r \in \{H', L'\}$. If both facilities have the same utility, i.e., $u_{ij1r}^X = u_{ij2r}^X$ for $i = 1, \dots, n$, $j = 1, \dots, m$, $r \in \{H', L'\}$, and the expected transport time is shorter for facility 1, i.e., $\delta_{ij1} < \delta_{ij2}$ for $i = 1, \dots, n$, $j = 1, \dots, m$, then $\nu_t(S^M(s(t)|s_j = D_1)) - \nu_t(S^M(s(t)|s_j = D_2)) \geq 0$ for all $t \geq 0$ and it is always better to deliver to the facility with the smaller expected service time.*

Proof. The claim will be shown by induction. First, note that the claim is trivially true for $t = 0$ since $\nu_t(s(0)) = 0$ for all states $s(0)$. We assume that $\nu_t(s1(t)) - \nu_t(s2(t)) \geq 0$ for all states $s1$ and $s2$ that are identical. Let MEDEVAC asset j^* be the asset that must transport casualties to a medical treatment facility. Let the state $s1(t+1) = S^M(s(t)|s_j = D_1)$ and let $s2(t+1) = S^M(s(t)|s_j = D_2)$. Next, after some rearranging:

$$\begin{aligned}
& \gamma(\nu_t(s1(t+1)) - \nu_t(s2(t+1))) = \\
& \sum_{k=1}^d \sum_{j=1, j \neq j^*}^m \sum_{i=1}^n (\delta_{ijk})^{-1} I_{\{s_j = D_k\}} (\nu_t(S^M(s1(t)|s1_j = 0)) - \nu_t(S^M(s2(t)|s2_j = 0))) \\
& + \sum_{i=1}^n \sum_{X=1}^N \sum_{R \in \{H, L\}} \lambda P_i^X P_{R|i}^X \left(\max_{j \in A_1(s1(t))} \{\nu_t(S^M(s1(t)|s1_j = (i, R, X)))\} - \max_{j \in A_1(s2(t))} \{\nu_t(S^M(s2(t)|s2_j = (i, R, X)))\} \right) \quad (4a) \\
& + \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H, L\}} \sum_{r \in \{H', L'\}} \frac{P_{r|R \cap i}}{\mu_{ij}^X} I_{\{s1_j = (i, R, X)\}} \left(\max_{D_k \in A_2(s1(t))} \{\nu_t(S^M(s1(t)|s_j = D_k)) + \gamma u_{ijk r}^X\} - \right. \\
& \quad \left. \max_{D_k \in A_2(s2(t))} \{\nu_t(S^M(s2(t)|s2_j = D_k)) + \gamma u_{ijk r}^X\} \right) \quad (4b) \\
& + \left(\gamma - \lambda - \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1, j \neq j^*}^m \sum_{R \in \{H, L\}} (\mu_{ij}^X)^{-1} I_{\{s_j = (i, R, X)\}} - \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^d (\delta_{ijk})^{-1} I_{\{s_j = D_k\}} \right) (\nu_t(s1(t)) - \nu_t(s2(t))) \\
& + ((\delta_{ij1})^{-1} - (\delta_{ij2})^{-1}) \nu(s(t+1)) - ((\delta_{ij1})^{-1} \nu(s1(t)) + (\delta_{ij2})^{-1} \nu(s2(t))).
\end{aligned}$$

As in Proposition 1, let $j^* = \arg \max_j \{A_1(s2(t))\}$. We can bound the expression above from below by setting both decisions in (4a) to j^* . Likewise, we can apply this same idea to the actions selected in both maximizations in (4b). Let $d^* = \arg \max_{D_k} \{A_2(s2(t))\}$, and set the destination in the first maximization to d^* . Moreover, the last line can be rearranged to yield:

$$(\delta_{ij1})^{-1} (\nu(s0(t)) - \nu(s1(t))) - (\delta_{ij2})^{-1} (\nu(s0(t)) - \nu(s2(t)))$$

after noting that $s_0(t) = S^M(s_1(t)|s_{1j} = 0) = S^M(s_2(t)|s_{2j} = 0)$. Moreover, we can bound this below by applying the induction assumption, with

$$(\delta_{ij1})^{-1}(\nu(s_0(t)) - \nu(s_1(t))) - (\delta_{ij2})^{-1}(\nu(s_0(t)) - \nu(s_2(t))) \geq ((\delta_{ij1})^{-1} - (\delta_{ij2})^{-1})(\nu(s_0(t)) - \nu(s_1(t))).$$

This yields:

$$\begin{aligned} & \gamma(\nu_t(s_1(t+1)) - \nu_t(s_2(t+1))) \geq \\ & \sum_{k=1}^d \sum_{j=1, j \neq j^*}^m \sum_{i=1}^n (\delta_{ijk})^{-1} I_{\{s_j = D_k\}} (\nu_t(S^M(s_1(t)|s_{1j} = 0)) - \nu_t(S^M(s_2(t)|s_{2j} = 0))) \\ & + \sum_{i=1}^n \sum_{X=1}^N \sum_{R \in \{H, L\}} \lambda P_i^X P_{R|i}^X \left(\nu_t(S^M(s_1(t)|s_{1j^*} = (i, R, X))) - \nu_t(S^M(s_2(t)|s_{2j^*} = (i, R, X))) \right) \\ & + \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1}^m \sum_{R \in \{H, L\}} \sum_{r \in \{H', L'\}} \frac{P_{r|R \cap i}}{\mu_{ij}^X} I_{\{s_{1j} = (i, R, X)\}} \left(\nu_t(S^M(s_1(t)|s_j = d^*)) - \nu_t(S^M(s_2(t)|s_{2j} = d^*)) \right) \\ & + \left(\gamma - \lambda - \sum_{i=1}^n \sum_{X=1}^N \sum_{j=1, j \neq j^*}^m \sum_{R \in \{H, L\}} (\mu_{ij}^X)^{-1} I_{\{s_j = (i, R, X)\}} - \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^d (\delta_{ijk})^{-1} I_{\{s_j = D_k\}} \right) (\nu_t(s_1(t)) - \nu_t(s_2(t))) \\ & + ((\delta_{ij1})^{-1} - (\delta_{ij2})^{-1})(\nu(s_0(t)) - \nu(s_1(t))). \end{aligned}$$

The first four lines are greater than or equal to zero by the induction assumption. The last line is greater than or equal to zero by Proposition 1 and by noting that $(\delta_{ij1})^{-1} - (\delta_{ij2})^{-1} \geq 0$. \square